Practice Exercises
for Math 6A Worksheets

Topics: Modules 1-4, Transformations, Add-Ons

Instructions: These exercises are optional and will not be graded. It is designed to help you prepare for the worksheets during discussion session. Questions that refer to a specific investigation (e.g. 1.2 #1) are in the workbook that is available online on Rational Reasoning.

1.1 Representational Equivalence

1. 1.1: #4, 6, 8 15
2. Rewrite the following expressions so that x, y, z each appear at most once and no terms are in the denominator. Assume that $x, y, z > 0$.
   (a). $\dfrac{x^2y^2(z^3)^3x}{x^4y}$
   (b). $\left(\dfrac{x^{11}x^{0.5}y^{\sqrt{7}}}{yz^3}\right)^0$
   (c). $x^{-1}\left(\dfrac{x^{0.5}x^4}{x}\right)^2$

1.2 Evaluating Formulas & 1.3 Relating Quantities and Solving Equations

1. 1.2: #3,4
2. 1.3: #1, 3, 5, 6, 7
3. Evaluate 3^0. Justify your answer completely – do not simply say "by a rule" or "by a calculator." You may justify by using one of the Law of Exponents that simplifies x^ax^b or $(x^a)^b$.
4. For what real numbers, x, do each of the following hold? Include all possible values for x and justify your answer completely.
   (a). $\sqrt{x^2} = x$?
   (b). $\sqrt{x^2} = -x$?
   (c). $|x - 1| = 4$?

2.1 Quantities & Co-variation of Quantities & 2.2 Changes in Quantities & Constant Rate of Change

1. 2.1 #10, 11
2. 2.2: #1, 2

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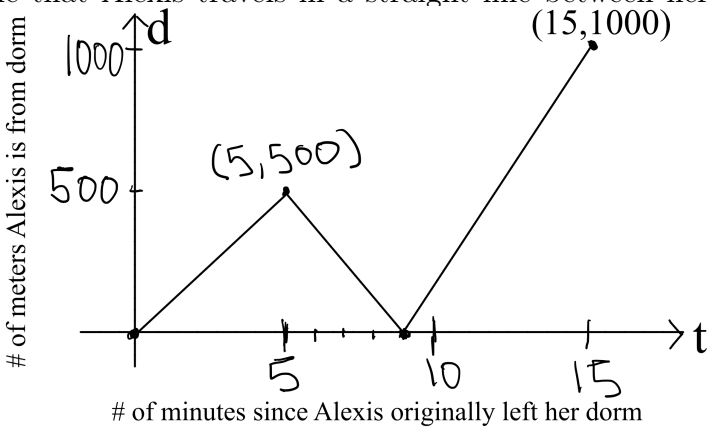
3. Alexis left her dorm and starts to walk to class, which is 1000 m away. A few minutes later, Alexis realizes she forgot her class notes. This graph represents Alexis' distance from her dorm in meters. Assume that Alexis travels in a straight line between her dorm and classroom.



   (a). How long does it take Alexis to realize her notebook is missing?

   (b). Does Alexis return to her dorm before going to class? If so, when? Justify.

   (c). What does the point $(15, 1000)$ represent? Be specific.

   (d). Is Alexis walking at a constant rate the entire trip? If not, over which time intervals, if any, is she traveling at a constant rate and what are her velocities?

   (e). On which leg(s) of the trip is Alexis' traveling fastest? Justify.

2.3 Constant Rate of Change and Linear Functions

1. 2.3: #2, 7

2. Given the information in this table:

x	-1	0	1
y	9	3	1
z	9	6	3

   (a) Do x and y vary at a constant rate of change? If so, justify by finding that rate.

   (b) Do x and z vary at a constant rate of change? If so, justify by finding that rate.

3. At 9 pm, Julie leaves home to go to the food store, Foodie, which is 2 miles from home. Assume she walks in a straight line from home to Foodie and she walks at a constant speed towards Foodie.

   (a) At 9:10 pm, Julie is 0.5 miles from home, how far is she from Foodie? *Depict.*

   (b) At 9:20 pm, how far is Julie from home? How far is Julie from Foodie? *Depict.*

   (c) Let t be the number of minutes since 9 pm. Let h be the number of miles Julie is away from home. If $\Delta t = 10$, what is Δh?

   (d) How fast is Julie walking? Include units.

2.4 Constant Rate of Change and Linearity

1. 2.4: #6

2. Jose is standing 3 feet from his mailbox when all of a sudden he begins walking away from the mailbox at a constant speed of 5 feet per second.

   (a). How far is Jose from the mailbox 2 seconds after he started walking away from it?

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(b). Write a formula that expresses Jose's distance from the mailbox (in feet), d, in terms of the number of seconds t since he started walking away from it.

(c). As Jose walks away from the mailbox, is his distance from the mailbox proportional to the time elapsed since he started walking away from the mailbox? Explain.

(d). As Jose walks away from the mailbox, is his *change in* distance from the mailbox proportional to the *change in* time that has elapsed since he started walking away from the mailbox? Explain.

3. Alexis goes on a road trip; her car gets 30 miles per gallon (mpg) and gas costs \$3.60 per gallon. Let n represent the number of miles Alexis has traveled since she started driving. *(similar to #9 on online homework 2.4)*

   (a). Suppose Alexis has driven for 300 miles on her road trip. How many gallons has she used? What is the cost of the gasoline that she used?

   (b). Write an expression in terms of n that represents the number of gallons of gasoline she has used since she started driving.

   (c). Write an expression in terms of n that represents the number of gallons of gasoline she has used since she started driving.

   (d). If her gas tank starts at full & holds 12 gallons, how many miles can she drive without stopping for gas? *Assume sh arrives at a gas station when she runs out.*

2.5 Exploring Average Speed

1. 2.5: #4, 8
2. A bus & car are stopped next to each other at a light. When the light turns green they start at the same time. Let t be the number of seconds since the light turned green.

   (a). The distance (in meters) the car has driven since the light turned green is given by $c = t^2 + 2t$. What is the car's average speed between $t = 2$ and $t = 5$ seconds?

   (b). The bus drives at a constant speed and travels 18 meters in the first 2 seconds. Which traveled further between $t = 2$ and $t = 5$ seconds? *Justify completely.*

   (c). Write an equation for the distance, d, the car is ahead of the bus after t seconds. At what time(s) is the distance zero?

   (d). The bicyclist is traveling at 4 m/sec. How far apart is the bus and the bicyclist after 5 seconds?

2^+ Solving Systems of Linear Equations

Solve each system of linear equations or show why there is no solution. When you find solutions, plug your answer(s) into the original equations to check your answer.

1. Assume the tortoise and hare (from question 2.4:#6) keep running until the hare catches the tortoise. After how many seconds does this happen?

2. **True or False**: A system of two linear equations in two unknowns always has exactly one solution. *Use a picture to help justify your answer. If it is true, explain why. If it is false, explain what else can happen.*

3. You have 25 coins in your pocket, consisting only of nickels and dimes. The total value of the coins is two dollars. How many nickels do you have and how many dimes do you have? *Show your work.*

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4. You manage *I Dream of Ice Cream,* an ice cream factory that specializes in 3 flavors: Vanilla, Chocolate, and Mint. Into each batch of Vanilla go 2 eggs and 2 cups of milk. Into each back of Chocolate go 3 eggs and 1 cup of milk. Into each batch of Mint go 1 egg and 1 cup of milk. You want to use all of your stock of 100 eggs and 60 cups of milk. Also, Mint is more popular than Vanilla and so you make twice as much Mint as Vanilla. How many batches of each flavor should you make? *Use variables V, C, & M for the number of batches of Vanilla, Chocolate, & Mint.*

5. Solve for x and y: $\begin{cases} x + y = 1 \\ x + 2y = 3 \end{cases}$

6. Solve for x, y and z: $\begin{cases} x + y + z = 1 \\ 2x + 2y + 2z = 1 \\ x - 2z = 0 \end{cases}$

7. Solve for x, y and z: $\begin{cases} x + y + 2z &= 1 \\ x - y - 2z &= 3 \\ y + z &= -4 \end{cases}$

8. Adam, Brenda, and Carl are running a race. Carl runs 1 yard per second slower than Adam and Brenda runs 10% faster than Adam. They each run for 10 seconds. With their distances combined, they run a total of 300 yards. How fast does each run? Include units in your final answer.
   Use the symbols A, B, and C as your variables. Define each variable carefully.

9. Find numbers a and b so that the system of equations

$$\begin{cases} 3x + y = 5 \\ x - ay = b \end{cases}$$

   has (i) no solutions, (ii) infinitely many solutions, and (iii) a unique solution at $(1, 2)$. Graph the two lines in all three situations.

2.6 The Distance Formula and the Equation of a Circle

1. 2.6: #1,2

2.7 Absolute Value

1. 2.7: #2, 7, 8, 10

3.1 Intro to Functions (Box Puzzle and Modeling Function Relationships)

1. 3.1: #1,2, 6

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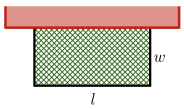
2. A farmer has 80 meters of fencing and would like to use the fencing to create a rectangular garden where one of the sides of the garden is against the side of a barn.



Let l represent the varying length of the rectangular garden (in meters). Answer the following questions keeping in mind that the quantities represent a real-world scenario and must make sense in this context.

   (a) Suppose the length of the garden is 30 meters. What is the width and area of the garden? Include units.

   (b) Write an expression for w in terms of l.

   (c) Write the area, A, of the garden as a function of the length of the garden, l. Express it as $A(l)$ to show that A is a function of l.

   (d) What is the domain of A?

   (e) The maximum area occurs when $l = 40$. What is the range of A?

3.2 Function Relations and Domain of Functions

1. 3.2: #1, 3, 8, 9, 10, 11, 12

2. At 10 am, Jules leaves home and walks to campus. At 7 pm, Jules returns home. Let t represent the time, in hours, since Jules left home at 10 am until she returns at 7 pm. Let d represent Jules' distance from home, in meters.

   (a) Is d a function of t (expressed as $d(t)$)? Justify your answer. If it is a function find the: domain, range, and $d(9)$.

   (b) Is t a function of d (expressed as $t(d)$)? Justify your answer. If it is a function find the: domain, range, and $t(0)$.

3. Let $f(x) = \sqrt{x-1}$.

   (a) What is the domain of f?

   (b) What is the range of f?

   (c) Evaluate $f(5)$.

4. Determine which statement is true and justify your answer with an example.

   (a) If two different input values have the same output value, then it is not a function.

   (b) If two different output values have the same input value, then it is not a function.

   (c) Both statements.

   (d) None of the above.

5. For each of the following relationships, described using different viewpoints of the Rule of Four, determine if b is a function of t (in which case it can be expressed as $b(t)$ instead of b). Explain why the relationship does or does not describe a function.

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(a) NUMERIC

(i).

b	0	1	2	3
t	5	2	4	5

(ii).

b	2	3	3	5
t	0	1	2	3

(iii).

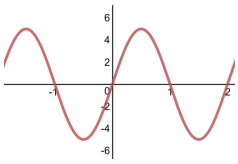
b	-5	-2	3	5
t	4	2	0	-3

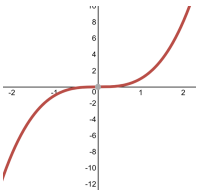
(b) VERBAL

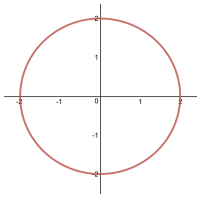
i. Bob walks at 4 miles per hour. Let t be the number of hours Bob has walked. Let b be the distance, in miles, Bob has walked after t hours.

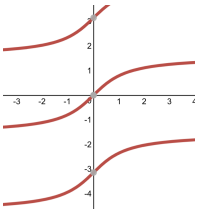
ii. Let t be the area of a triangle with base b.

(c) VISUAL In these graphs, t-values are on the horizontal axis and b values are on the vertical axis.

(i). 

(ii). 

(iii). 

(iv). 

(d) ALGEBRAIC

(i). t^2 (ii). $b = 3t + 5$ (iii). $t^2 = b$ (iv). $b = \sqrt{t}$

3.3 Using and Interpreting Function Notation

1. 3.3: #5
2. Let $a(x) = 4x, b(x) = 4|x|$, $c(x) = \sqrt{(4x)^2}$, and $d(x) = (\sqrt{4x})^2$.
   (a). Evaluate each of these functions at 1 and -1 or state why you cannot.
   (b). What is the domain of each function?
   (c). Are any of these functions equivalent (i.e., the same)? If so, which?

3.4 Function Composition in context

1. 3.4: #2, 3, 6, 7, 8

3.5 Function Composition (Reversal)

1. 3.5: #5, 6

3.6 Inverse Functions

1. 3.6: #5, 7-10,12-14
2. The volume of a sphere (in cm^3) can be expressed as function of the radius of the sphere (in cm) as follows: $V = f(r) = \frac{4}{3}\pi r^3$.
   (i). **Verbally interpret** what $f^{-1}(100)$ means physically (not algebraically).
   (ii). Find the value of $f^{-1}(100)$. Include units.
   (iii). Does $f(r)$ have an inverse function? If so, find $f^{-1}(V)$. If not, why not?

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(iv). Suppose the radius of the sphere is increasing at 2 cm per minute t minutes after noon today. Let $g(t)$ define the function whose input is time (in minutes) since noon today and whose output is radius of the sphere (in cm). Find $g(t)$.

(v). In the context of this question, does $f(g(t))$ make sense? If so, what does it mean?

(vi). In the context of this question, does $g(f(r))$ make sense? If so, what does it mean?

3^+ Transformations: Stretch, shift, flip

Tip: Use a graphing tool to help you solidify your understanding of how the graphical and algebraic expressions for transformations of functions relate to each other. Then try performing transformations without a graphing tool to check your understanding. Check your work! You may find this Desmos Module useful: www.desmos.com/calculator/9dzgyvx6en

1. These questions are about a general function, $f(x)$, but you can consider the specific function first to help you visualize this, then generalize to any function. For example, consider $f(x) = x^2$ for $-1 < x \leq 2$.

   (a). What transformations were done to go from $f(x)$ to $2f(x) + 3$?
   List them in the order in which they were done and be specific.

   (b). What transformations were done to go from $f(x)$ to $-f(x-1) - 4$?
   List them in the order in which they were done and be specific.

   (c). Suppose $A(x)$ is $f(x)$ first shifted to the right by 2, then flipped across the y-axis. Suppose $B(x)$ is $f(x)$ first flipped across the y-axis, then shifted to the right by 2. Does $A(x) = B(x)$? Equivalently, does the order of these transformations matter? **Justify.**

2. Let $f(x) = x^2$ for $-1 < x \leq 2$. Graph $f(x)$, then depict these transformations of f **graphically** and **algebraically**. For this exercise, apply only 1 at a time transformation – do not combine them.

   (a) Vertical Shift of f: up by 1

   (b) Horizontal Shift of f: left by 2

   (c) Vertical Stretch of f: by 3

   (d) Horizontal Stretch of f: by 4 *(so that 1 stretches to 4)*

   (e) Vertical Flip of f

   (f) Horizontal Flip of f

3. Want more practice? Repeat the previous exercises by:

   (a) changing the numbers and direction of shift (eg up/down or left/right).

   (b) changing the function f

   (c) combining transformations

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4.0 Relevant Skills and Procedures for Starting M4 & 4.1 The Meaning of Exponents

1. 4.0: # 1-3, 7-10, 16
2. 4.1: #7, 9, 10
3. Use properties of exponents to answer these questions.
   Assume that $b > 0$ and $b \neq 1$.
   (a). Suppose $2^3 = 2^{2x}$. What does x equal?
   (b). Suppose $b^x b^2 = (b^x)^4$. What does x equal?

4.2 Comparing linear and exponential functions

1. 4.2: # 6,7
2. The following table is a few values of a function:

x	0	1	2	3
f(x)	2	6		

   (1). Complete the table assuming that f is a linear function.

x	0	1	2	3
f(x)	2	6		

   (2). If f is a linear function, what is the equation for f?

   (3).Complete the table assuming that f is an exponential function.

x	0	1	2	3
f(x)	2	6		

   (2). If f is an exponential function, what is the equation for f?

4.3 1-unit Scale (Growth/Decay) Factors, % Change, & Initial Values

1. 4.3: #1, 2, 5, 6, 11, 14
2. Suppose you opened a bank account with $100 on January 1, 2015. The account earns 3% interest each year, which is deposited into the account, and no additional money is added to the account.
   (a). How much money is in the account on January 1, 2016?
   (b). How much money is in the account t years after January 1, 2015?
   (c). How many years does it take to have $150 in your account?

4^+ Motivate e. Graph Exponential Functions & their Inverses

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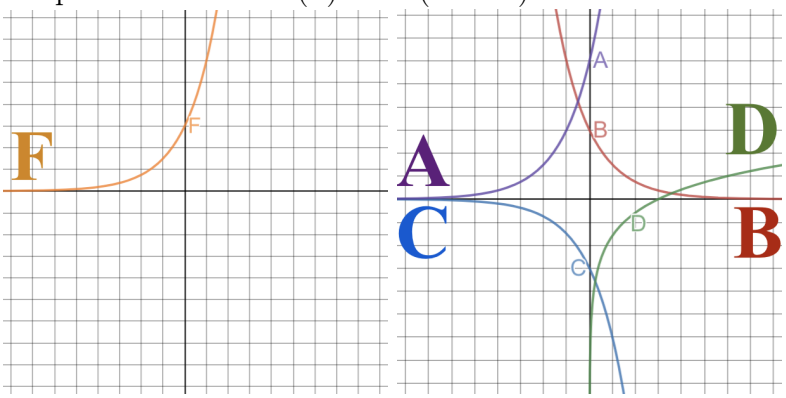
1. Graphed below are $F(x) = b^x$ (on left) and transformations of F (A,B,C,D on right).



   (a) What can you say about b? Write an inequality that describes the size of b.

   (b) The graphs of A, B, C, and D are transformations of F (which can include a flip across $y = x$). Write equations for A, B, C, and D in terms of F.

2. Without a graphing tool, graph $f(x) = 2^x$ and $g(x) = 0.5^x$ on the same axes. Plot at least 3 points on each graph. What can you say about how f and g relate?

3. Without a graphing tool, graph $h(x) = 5(2^x)$. Plot at least 3 points on its graph. Does $h(x) = 10^x$? Justify.

4.8 The Inverse of an Exponential Function

1. 4.8: #1, 2, 8
2. Properties of logarithms can be derived from properties of exponents. This question is designed to help you understand logarithms more. Recall if $f(x) = b^x$, then $f^{-1}(x) = \log_b(x)$ as $\log_b(x)$ undoes b^x and vice versa.

   Some Properties of Logarithms:
   Let b, c, m, n be constants. Assume $m, n, b > 0$ and $b \neq 1$.
   (1). $\log_b(mn) = \log_b(m) + \log_b(n)$
   (2). $c \log_b(m) = \log_b(m^c)$.

3. You can use properties of logarithms to help answer these questions.

   (a). Justify why $\log_{10}(x + y) \neq \log_{10}(x) + \log_{10}(y)$.

   (b). Justify why $\log_{10}(x) \log_{10}(y) \neq \log_{10}(xy)$.

   (c). Solve for x in the equation $6 = 2(10^x)$. *Your answer may contain a logarithm.*

   Note: Statements (1) & (2) still hold if we change from base 10 to any other base. For instance, $\log_3(x + y) \neq \log_3(x) + \log_3(y)$.

4.9 Solving Logarithmic Equations

1. 4.9: #1, 2, 4

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