Theorem 3. Special Limits

Let c be a real number in the domain of the given function and let n be a positive integer. The following limits hold:

1. $\lim_{x\to c} \sin x = \sin c$
2. $\lim_{x\to c} \cos x = \cos c$
3. $\lim_{x\to c} \tan x = \tan c$
4. $\lim_{x\to c} \csc x = \csc c$
5. $\lim_{x\to c} \sec x = \sec c$
6. $\lim_{x\to c} \cot x = \cot c$
7. $\lim_{x\to c} a^x = a^c \ (a > 0)$
8. $\lim_{x\to c} \ln x = \ln c$
9. $\lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$
10. $\lim_{x\to 0} (1 + x)^{\frac{1}{x}} = e$
11. $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
12. $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

Theorem 4. Squeeze Theorem

Let f, g and h be functions on an open interval I containing c such that for all x in I,

$$f(x) \le g(x) \le h(x).$$

If $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$,

then $\lim_{x\to c} g(x) = L$.

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Example Evaluating limits analytically

Evaluate the following limits.

1. $\lim_{x\to\pi} \cos x$
2. $\lim_{x\to 3} (\sec^2 x - \tan^2 x)$
3. $\lim_{x\to\pi/2} \cos x \sin x$
4. $\lim_{x\to 1} e^{\ln x}$
5. $\lim_{x\to 0} \frac{\sin x}{x}$

① $\lim_{x\to\pi} \cos x = \cos\pi = -1$

(∵ using $1 + \tan^2 x = \sec^2 x$)

② $\lim_{x\to 3} (\sec^2 x - \tan^2 x) = \lim_{x\to 3} 1 \Rightarrow 1$

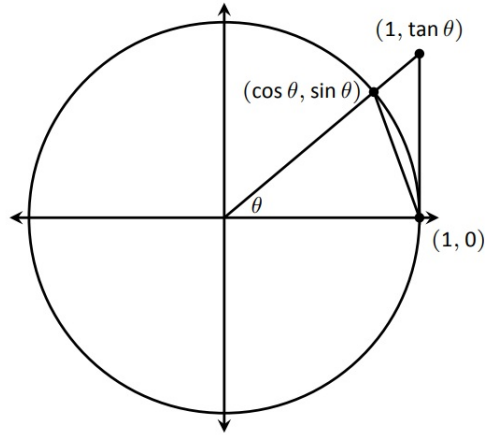
③ $\lim_{x\to \pi/2} \cos x \sin x = \cos\frac{\pi}{2} \sin\frac{\pi}{2} \Rightarrow (0)(1) = 0$

④ $\lim_{x\to 1} e^{\ln x} = e^{\ln(1)} = e^0 = 1$

⑤ $\lim_{x\to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$

"Indeterminant form"
we use squeez theorem to solve this $\lim_{x\to 0} \frac{\sin x}{x}$

Example $\lim_{x \to 0} \frac{\sin x}{x}$ **(Using Squeeze Theorem)**

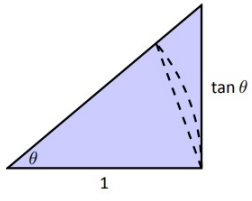


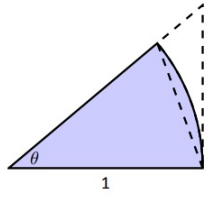
The unit circle and related triangles.

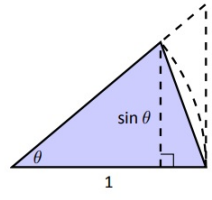
So area of small triangle = $\frac{1}{2}\sin\theta$

area of big triangle = $\frac{1}{2}\tan\theta$

area of sector = $\theta/2$







$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

Multiply by $\frac{2}{\sin\theta}$

$$\frac{\tan\theta}{\sin\theta} \geqslant \frac{\theta}{\sin\theta} > 1$$

$$\frac{1}{\cos\theta} \geqslant \frac{\theta}{\sin\theta} \geqslant 1$$

As we know that

area of Triangle = $\frac{1}{2}bh$

area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

unit circle, $r = 1$

taking Reciprocal of values

$$\cos\theta \le \frac{\sin\theta}{\theta} \le 1$$

taking limits

$$\lim_{\theta\to 0} \cos\theta \le \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \le \lim_{\theta\to 0} 1$$

$$\cos 0 \le \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \le 1$$

$(\because \cos 0 = 1)$

$$1 \le \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \le 1$$

"using squeez theorem"

If $f(x) \le g(x) \le h(x)$

and $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = L$

then $\lim_{x\to 0} g(x) = L$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

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